

Is there a contradiction between deductive reasoning
and Descartes' claim for the priority of the infinite?

Descartes proves the existence of God in the Third Meditation with an argument that relies on the metaphysical priority of the infinite to the finite. Deductive reasoning of finite numbers to an understanding of something limitless brings into question the validity of the metaphysical claim. This paper will explain Descartes' argument of the priority of the infinite, and then analyze his definitions of “infinite” and “deduction” to demonstrate that there is not a contradiction between the ability to deduce the limitlessness of the real numbers and the metaphysical priority of the infinite.

Descartes demonstrates the metaphysical priority of the infinite through his hierarchical model of reality (Third meditation). In this model, he first distinguishes between the two types of reality: “objective reality”—the idea of a thing, and “formal reality”—the thing itself. A simple way to understand this is through an analogy of a photograph. A photograph captures the idea of a subject such that the photograph has objective reality and the subject of the photograph has formal reality. Crucially, a photograph cannot capture all of the information contained in the subject of the photograph, including the true colors, sizes, resolutions, and all other aspects of something three-dimensional that can in no way be accounted for in a mere two-dimensional representation. Through this analogy, it is clear that something with objective reality is a reduction of something with formal reality that comes prior, or as Descartes writes, “all the intricacy which is contained in the idea merely objectively—as in a picture—must be contained in its cause, whatever kind of cause it turns out to be; and it must be contained not merely objectively or representatively, but in actual reality, either formal or eminently” (Principles Part 1, 17, p. 198).

Under each branch—Objective and Formal—there are further degrees of reality in this hierarchical model. Something can have a higher degree of objective reality than something else with objective reality in the same way that something can have a higher degree of formal reality than something else with formal reality—the metric being one of causality. As Descartes explains, “...something cannot arise from nothing, and also that what is more perfect—that is, contains in itself more reality—cannot arise from what is less perfect” (28). In the case of formal reality, this could be understood through an example of a red brick. While both the brick and the color red of the brick have formal reality, the brick is said to have *more* formal reality because its existence does not depend on the color red in the way that the color red depends on the existence of the brick. Similarly, in the case of objective reality, the *idea* of the brick is said to have more objective reality than the *idea* of the color red of the brick for the same causal reason.

With this model of reality, Descartes demonstrates the metaphysical priority of the infinite to the finite. He states “the idea that gives me my understanding of a supreme God, eternal, infinite, <immutable,> omniscient, omnipotent, and the creator of all things that exist apart from him, certainly has in it more objective reality than the ideas that represent finite substances” (28). In other words, as finite beings, we cannot imagine something more perfect than we are—that would be equivalent to taking a photograph of something with truer color, resolution, size, and more dimensions than subject than the subject itself, or equivalent to the color red causing the brick to exist—both of which are impossible.

While the priority of the infinite is clear within this hierarchical model of reality, it becomes less obvious if you actually test it. For example, it seems that you can understand the infinite through only your understanding of the finite by summing up finite numbers until

the real numbers are revealed to be limitless. This process of summing up finite numbers to reveal a limitless notion is considered deduction, which Descartes legitimizes throughout the meditations as a process one can use to arrive at a clear and distinct understanding. Therefore, if ‘being limitless’ is all Descartes means infinity to be, then it proves to be deducible from only a finite understanding, debunking the metaphysical priority of the infinite to the finite. This means that either the definition of infinity as limitless is insufficient—what we deduce is something entirely different—or that the method of deduction relies on some innate knowledge given to us prior. The following sections will explore both notions, starting with the definition of the infinite and then moving to the definition of deduction.

The definition of the infinite has shown to be crucial to this proof, yet Descartes provides no formal definition anywhere in the Third Meditation. Instead, he defines it in writings after the fact in replies to objections as well as in letters. In the first set of replies to objections raised about the Third Meditation, Descartes provides this formal definition:

“Now I make a distinction here between the *indefinite* and the *infinite*. I apply the term ‘infinite’, in the strict sense, only to that in which no limit of any kind can be found; and in this sense, only God alone is infinite. But in cases like extension of imaginary space, or the set of numbers, or the divisibility of the parts of a quantity, there is merely some respect in which I do not recognize a limit; so here I use the term ‘indefinite’ rather than ‘infinite’, because these items are not limitless in every respect.” (CSMC II, 81)

Here, the distinction between indefinite and infinite is one of negation versus affirmation: indefinite is understood through the negation of a limit as recognized *by* someone, while the infinite (in the strict sense) is understood through the positive affirmation of limitlessness, independent of anyone’s recognition. However, it is unclear what the “strict sense” means, and whether there is a corresponding “weak sense” that required Descartes to make this specification. It can be read that the strict sense is the *thing* that is infinite and the weak sense is the *attribute* of

infinite, given Descartes distinction, “in the case of infinity, even if we understand it to be positive in the highest degree, nevertheless our understanding of it is negative because it depends on our not noticing any limitation in the thing...but in the case of the thing *itself*,...our understanding is positive” (CSMMS II, 81). This calls for a modification of the earlier distinction of the indefinite and the infinite: the indefinite and the “weak” infinite are understood through the negation of a limit as recognized by someone, while the “strict” infinite is understood through the positive affirmation of limitlessness.

This reading of the “strict” and “weak” sense of the infinite can appear at first glance to undermine the need for the “indefinite” by conflating the “weak” infinite with the indefinite. Although they are both understood through negation, it is not to say that they are therefore the same attribute. Specifically, the “weak” infinite is only understood through negation because of our inability to fully grasp the attribute of infinity by virtue of our being finite, while the indefinite is understood through negation regardless of whether we were finite or infinite beings. For example, if we take one of the examples of indefinite quantities that Descartes provides in the definition—the set of real numbers—it is in its definition to always have one more number. In other words, even an infinite being cannot see the end of the real numbers. On the contrary, a “weak” infinite attribute would be positively limitless to an infinite being. Therefore, we can now re-state the distinction between the “strict” infinite, the “weak” infinite, and the indefinite more precisely: The “strict” infinite is the *thing* which can be understood as positively limitless; The “weak” infinite is the *attribute* which can be understood by finite beings through negation, but through infinite beings positively; The indefinite is the attribute defined as the negation of a limit and understood as such by finite and infinite beings alike.

Using this reading of the three terms, it seems that the contradiction between deduction and the priority of the infinite is immediately resolvable—we merely deduce the indefinite and not the “weak” infinite when we realize that the real numbers are limitless. However, the question still remains whether it is possible for us to infer the “weak” or “strong” infinite from our understanding of the indefinite. In order to see whether this is possible, it is crucial to understand the relationship between the indefinite and both the “weak” and “strong” infinite, which Descartes gives in two places—a thought experiment in the Third Meditation and a conversation with Burman—both of which deal with the jump from the indefinite to the infinite:

“...even if my knowledge always increases more and more, I recognize that it will never actually be infinite, since it will never reach the point where it is not capable of a further increase; God on the other hand, I take to be actually infinite, so that nothing can be added to this perfection..’ (CSMC II, 32)

“Now, if it were I who had given myself my nature and make-up, I would have given myself all the perfections of God. I think I would have given myself these perfections in accordance with my indefinite conception of them. For example, I would have given myself greater knowledge than I now possess; and when I had that greater knowledge, I would then have given myself greater knowledge still, and so on. Now when indefinites are multiplied in this way, they become infinite; or rather they become the infinite, since the infinite is the same as the indefinite multiplied in this way. As I increased my knowledge more and more in this way, I would by the same token have increased my other attributes, and I would end up as God.” (CSMC III, 338)

In both quotes, Descartes is explaining the thought experiment of someone who increases their knowledge in a recursive process such that each time they gain more knowledge, they have more knowledge with which to increase their knowledge again, creating an exponential process of increasing knowledge. While the thought experiments are both the same, Descartes arrives at opposite conclusions about the relationship between the indefinite to the infinite.

Specifically, in the first quote, he arrives at the conclusion that increasing knowledge of indefiniteness more and more will never reach the infinite because something defined as the

negation of a recognizing a limit cannot somehow become something that is defined to positively be limitless in all respects. For example, it does not seem plausible that you can multiply the set of real numbers until it is positively limitless. Even if somehow you were able to think of the real numbers in infinite dimensions, you would just have an infinite amount of still indefinite sets. On the other hand, in the second quote, he arrives at the conclusion that increasing knowledge of indefiniteness more and more will reach a point where no more knowledge can be attained and the “strict” infinite is reached. The “strict” infinite, specifically, is reached because of Descartes subtle word change when he writes, “now when indefinites are multiplied in this way, they become infinite; or rather, they become *the* infinite... I would end up as God.” This indicates that multiplying indefinites gives the *thing* which is infinite.

While it seems that Descartes is himself confused about the relationship between the indefinite and the infinite, this is in fact, not the case. In the conversation with Burman, we can read Descartes to have made a small error in word-choice by using the term indefinite when he was actually speaking about the “weak” infinite. Specifically, he says, “...we understand the perfections and attributes of God, but we do not conceive of them—or, rather, in order to conceive of them, we conceive of them as indefinite...if I had given myself my nature and make-up, I would have given myself these perfections in accordance with my indefinite conceptions of them.” Clearly, he is talking about the “weak” infinite because he is referring to the “perfections” and “attributes” of God which are positively infinite, but only understood by a finite being through negation. If we read him to have simply misspoken by saying indefinite when he meant the attribute of “weak” infinite, then there is no contradiction at all between the two quotes, but instead, one coherent reading. The reading is that the indefinite cannot become the “strict” infinite, but an understanding of the “weak” infinite can give the “strict” infinite as

the being gains knowledge exponentially in order to understand how to view the “weak” infinite in the positive way that does not rely upon negation.

This shows that the infinite does not *come from* the indefinite, but says nothing about its ability to be *inferred* from the indefinite—the question remains whether we can understand the infinite from our finite understanding. I argue that even though the infinite does not arise out of the indefinite, we can still infer it from the indefinite as Descartes does in two places: the second set of replies to objections on the Third Meditation and a letter to Chanut. In the reply, Descartes shows how the “strict” infinite can be inferred from the indefinite when he writes, “..when I count, I cannot reach a largest number, and hence I recognize that there is something in the process of counting which exceeds my powers. I contend that from this alone, not that an infinite number exist, but that I have the power of conceiving that there is a thinkable number which is larger than any number that I can ever think of, and hence that this power is something which I have received not from myself but from some other being which is more perfect than I am.” In this way, Descartes shows that you can infer the existence of something truly infinite through an understanding of the limitlessness of the real numbers.

In the letter to Chanut, he shows how the “weak” infinite can be inferred from the indefinite through his discussion of the eternal duration of the universe as understood through an inference from the indefinite understanding of extension: “If we consider the extension of the world in this way and then compare it with its duration, it seems to me that the only thought it occasions is that there is no imaginable time before the creation of the world in which God could have created it if he willed...I do not think that we have any grounds for concluding that he really did create if an indefinitely long time ago” (CSMC III, 329). Here, Descartes makes an inference to eternity, which can be read as the “weak” infinite, from understanding the universe

through indefinite extension. Therefore, in both examples, Descartes shows that an understanding of the indefinite as a negation of a limit can motivate questions about whether something positively limitless exists. It does not matter that we contemplate this being or what the attribute of infinity would entail—all that matters is that we acknowledge that something *could* exist that is positively limitless from an understanding of the negation of a limit.

In reading Descartes definition of the infinite using the three terms of the indefinite, “weak” infinite, and “strict” infinite, it becomes clear that we deduce—not the “weak” or “strict” infinite—but the indefinite from realizing that the real numbers are limitless. However, from that understanding of the indefinite, we can take one more step to infer the “weak” or “strict” infinite by asking the question of what it would mean for something to be positively limitless rather than a negation of a limit, as Descartes did in the reply to objections as well as in the letter to Chanut. In this way, it is still unclear whether there is a contradiction between deduction and the metaphysical claim that the infinite is prior to the finite, requiring us to understand Descartes’ definition of deduction next.

Descartes defines deduction most clearly in his Rules for the Direction of the Mind as the “inference of something as following necessarily from other propositions which are known with certainty” (CSMC III, 15). In the case of the real numbers, when you start counting: 2,3,4,..The numbers of 2,3,and 4... are all propositions that are known with certainty upon inferring from the previous number that you can add one to it. Continuing this process and remembering all of the prior propositions of the numbers inferred from before allows you to make the final inference: the real numbers can always be continued by adding one to a number. Therefore, because this process is one involving multiple steps of inference from adding one to each number, it can be considered deduction.

Moreover, it parallels the example of deduction that Descartes provides with his example of a long chain with an ungraspable amount of intermediate links. He explains, “even if we cannot take in at one glance all the intermediate links on which the connection depends, we can have knowledge of the connection provided we survey the links one after the other, and keep in mind that each link from first to last is attached to its neighbor” (CSMC, 15). In this way, the real numbers can be seen as an analogy to a long chain of intermediate links in which each real number is a link and observing each one by one allows you to see the “end” or “limitlessness” of the chain, respectively.

Crucial to this process, however, is “intuition,” or the “indubitable conception of a clear and attentive mind which proceeds solely from the light of reason” (CSMC 14). The important part of this definition is that intuition comes from the “light of reason,” which can be read as the innate knowledge that is within us prior to any empirical investigation or proof. For example, the assertion ‘I am thinking, therefore I am, or I exist,’ is self-evident, not deduced because deduction would rely on the premise ‘Everything which thinks, exists.’, which we have no way of deducing (CSMC,100). If the process of deduction relies upon a chain of innate inferences that we hold within ourselves prior to any experiences, then there is no contradiction between the method of deduction to arrive at the “indefinite” and subsequently the “weak” and “strict” infinite and the metaphysical priority of the infinite—the method of deduction that arrives at the indefinite and the inference that comes after to arrive at the infinite is merely a way of making known the prior knowledge that is already within us of the infinite.

This is a compelling reading of deduction given Descartes adherence to the Socratic view of learning as he explains in a Letter to Voetorius (CSMC III, 232). His defense for this view of learning lies in the idea that when we first conceive of a triangle as children, we do not have to

understand all of the properties in order to identify that what is before us is a triangle. Neither do we come to identify that it is a triangle through the senses because the true conception of the triangle is independent of the visual-aid that represents it. Specifically, the independence is shown through the fact that any drawn triangle that we identify with three straight lines is actually defective and more accurately, it is composed of curvy lines when truly observe it. Because the drawing does not capture the three straight lines of the triangle, but we nevertheless are able to identify that it is a triangle, Descartes concludes that the conception does not come about from the senses (CSMC II, 262). If it is not taught, nor sensed, then it must be innate. In this way, Descartes adheres to the Socratic view of learning in which one remembers what is already innate through the process of deduction that *explicitly* makes known the infinite that was *implicitly* known prior (338).

Therefore, if we read Descartes definitions for the indefinite—the attribute that is defined as the negation of a limit, the “weak” infinite—the attribute that is understood as the negation of a limit by finite beings, but as positively limitless by infinite beings, and the “strict” infinite—the thing that is positively limitless in all respects, it provides one coherent reading of Descartes’ arguments. Specifically, with this reading, we can see why the real numbers that we can easily recognize to be limitlessness does not equate to “infinity,” but merely the indefinite attribute that is *always* defined as the negation of a limit. Then, we can see how our understanding of the indefinite can nevertheless facilitate our inference to the “weak” and “strict” infinite. Yet, this is not at all a contradiction with the metaphysical priority of the infinite that Descartes’ outlines in his structure of reality because the deduction and inference that is performed for us to arrive at an understanding of the infinite is merely the remembering of the innate ideas contained within us prior.